Journal of Engineering Physics and Thermophysics, Vol. 72, No. 4, 1999

SIMULATION OF CORRELATION OF SHEAR RATES WITH PRESSURE FLUCTUATIONS IN THE EQUATION FOR REYNOLDS STRESSES

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UDC 532.517.4

A simulation of the correlation that contains pressure fluctuations and determines the process of redistribution of energy between the components of the tensor of Reynolds stresses is made. The results obtained can be used to construct new models of second-order turbulence.

Introduction. The solution of the applied problems of hydrodynamics and heat and mass transfer in turbulent flows of liquids and gases requires a reliable and universal enough model of turbulence. Its construction is usually based on the equation for the second one-point moments:

$$\frac{DR_{ij}}{Dt} = F_{ij} + P_{ij} + \Phi_{ij} - 2\varepsilon_{ij} + D_{ij} \,. \tag{1}$$

In Eq. (1), convective transfer and the processes of production (F_{ij} and P_{ij}) are described exactly. The terms that contain correlations of pressure fluctuations (Φ_{ij}) and dissipation (ε_{ij}) and diffusion (D_{ij}) terms must be simulated, since they contain unknown higher-order correlations. In [1] an analysis of investigations that deal with the simulation of unknown correlations was made; from the results of this analysis the following conclusions were drawn:

1) to increase the accuracy and universality of the second-order turbulence models more accurate models are needed for the correlations Φ_{ij} and ε_{ij};

2) at the present time there are no procedures for direct measurements of these correlations; therefore the available experimental data cannot be used to refine the available approximations;

3) to simulate unknown correlations, it is advisable to use the results of direct numerical simulation of the Navier–Stokes nonstationary equations.

The aim of the present work is to determine the coefficients and form of approximations for the correlation Φ_{ij} that describes the process of the redistribution of energy between the components of the tensor of Reynolds stresses. In contrast to earlier published investigations the form of approximations and empirical coefficients were found by means of direct comparison with the already available data of numerical solution of the Navier–Stokes nonstationary equations, i.e., by direct numerical simulation (DNS). Attention was mainly paid to the search for the means of correct accounting for all the main factors that influenced the process of redistribution of the energy of turbulence via pressure pulsations irrespective of the complexity of the relations obtained. It was assumed that, if need be, the approximations obtained could be used as a basis for deriving simpler equations.

1. Models for the Correlation of $\Phi_{(2)ij}$. When performing simulation, the correlation $\Phi_{ij} = (\langle pu_{i,j}\rangle + \langle pu_{j,i}\rangle)/\rho$ is usually represented as a sum of three terms:

$$\Phi_{ij} = \Phi_{(1)ij} + \Phi_{(2)ij} + \Phi_{(3)ij} \,, \tag{2}$$

where $\Phi_{(1)ij}$ depends only on the interaction of velocity fluctuations among themselves and reflects the fact that their field approaches an isotropic state. In [1] the simulation of the term $\Phi_{(1)ij}$ was made, as a result of which the following approximation was obtained:

Institute of Physics of the Latvian University, Salaspils. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 4, pp. 663-671, July-August, 1999. Original article submitted February 10, 1998.

$$\frac{\Phi_{(1)ij}}{\varepsilon} = -C_1 b_{ij} + C_2\left(b_{ij}^2 + \frac{2}{3} II\delta_{ij}\right), \tag{3}$$

$$C_1 = C_{10f} + C_{11f}F, \quad C_2 = 3C_{10f}, \quad C_{10f} = 1.2, \quad C_{11f} = 4.28,$$

$\Phi_{(2)ij}$ depends on the interaction of the mean shift in the velocity with velocity fluctuations, and $\Phi_{(3)ij}$ takes the effect of the wall into account.

Detailed surveys of the works and description of the technique used to simulate the term $\Phi_{(2)ij}$ are published in [2-4]. In simulation of this term, it is usually assumed that the gradient of averaged velocity can be taken outside the integral in formal solution of the Poisson equation. The results of direct numerical simulation [5] showed that this assumption is satisfied approximately for a shear flow in a plane channel. In this case the expression for the unknown term is sought in the form

$$\Phi_{(2)ij} = 4U_{p,q}(M_{ijpq} + M_{qjpi}),$$

where M_{ijpq} is a fourth-order tensor, which can be expressed in the form of a series in the degrees of the anisotropy tensor of the Reynolds stresses b_{ij}. It is shown that the most general form of this dependence is the quadratic one. In this case in the approximation for $\Phi_{(2)ij}$ 15 coefficients appear which must be determined from experimental data. To minimize the number of empirical coefficients and to calibrate them, additional conditions were used in [2], which were based on the first principles. These are: tensor invariance, field-form invariance, feasibility of the model, etc. With allowance for these principles, the approximation for $\Phi_{(2)ij}$ can be written in a general form as

$$\frac{\Phi_{(2)ij}}{\varepsilon} = 4\frac{K}{\varepsilon}\left[d_1 S_{ij} + d_2\left(b_{ip}S_{pj} + b_{jp}S_{pi} - \frac{2}{3}\langle bS\rangle\delta_{ij}\right) + d_3(b_{ip}W_{pj} + b_{jp}W_{pi}) + \right.$$
$$+ d_4\langle bS\rangle\left(b_{ij}^2 + \frac{2}{3}II\delta_{ij}\right) + d_4\langle b^2S\rangle b_{ij} + d_5\langle bS\rangle b_{ij} +$$
$$\left. + d_6\left(b_{ip}^2 S_{pj} + b_{jp}^2 S_{pi} - \frac{2}{3}\langle b^2 S\rangle\delta_{ij}\right) + d_7(b_{ip}^2 W_{pj} + b_{jp}^2 W_{pi}) + d_8(b_{ip}W_{pq}b_{qj}^2 + b_{jp}W_{pq}b_{qi}^2)\right], \tag{4}$$

where $S_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i})$; $\langle bS\rangle = b_{ij}S_{ji} = -\frac{1}{2}\frac{P}{K}$; $W_{ij} = \frac{1}{2}(U_{i,j} - U_{j,i})$; $\langle b^2S\rangle = b_{pq}b_{qm}S_{mp}$, and the coefficients d_i depend on the scalar invariants II and III.

The most well-known approximations for $\Phi_{(2)ij}/\varepsilon$ employ only several first terms of expansion (4):

the model of [6]: $d_4 = d_5 = d_6 = d_7 = d_8 = 0$, $4d_1 = 0.8$, $4d_2 = 1.75$, $4d_3 = -1.31$;

the model of [7]: $d_4 = d_5 = d_6 = d_7 = d_8 = 0$, $4d_1 = C_3 - C_3(-2\,II)^{1/2}$, $4d_2 = 1.25$, $4d_3 = -0.4$, $C_3 = 0.8$, $C_3^* = 1.3$;

the model of [2] contains cumbersome dependences of d_i on the invariants II and III.

The authors of [2] showed that after the use of the conditions of invariance and feasibility, only seven of the above-mentioned 15 coefficients are to be found experimentally. Experimental data and the results of direct numerical simulation were used to determine them. This is accomplished by comparing the results of calculations of $\Phi_{(2)ij}$ by formula (4) with experimental data for the difference $(\Phi_{ij} - \Phi_{(1)ij})$, where

$$\Phi_{(1)ij} = -(2 + 15.5F^{1/2})\,\varepsilon b_{ij}. \tag{5}$$

In making this indirect comparison all the uncertainties in the description of the term $\Phi_{(1)ij}$ by formula (5) exert a direct effect on the magnitude of the coefficients d_i and on the accuracy of the determination of $\Phi_{(2)ij}$. Earlier [1] it was shown that relation (5) describes DNS data with a large uncertainty [5]. Therefore, it should be expected that the coefficients presented in [2] will need a more precise definition.

The appearance of the data of direct numerical simulation of channel flow radically changes the situation, since they can be used to find the coefficients of Eq. (4) by comparing directly the DNS data for $\Phi_{(2)ij}$ with

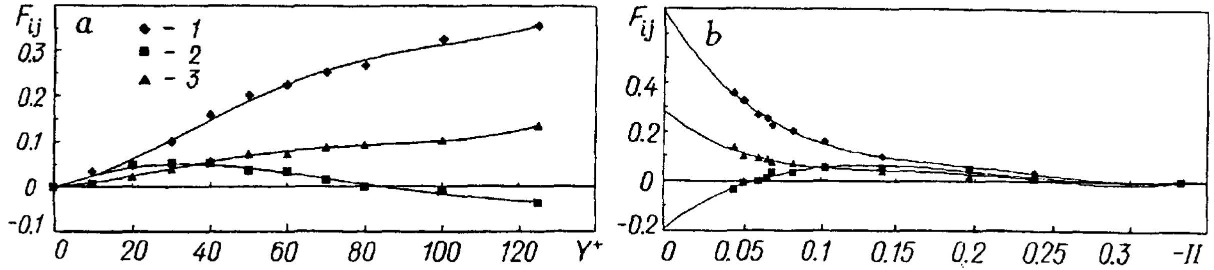

Fig. 1. DNS data for the components of the tensor F_{ij} (a) and the dependence of these components on invariant II (b): 1) F_{xx}, 2) F_{yy}, 3) F_{xy}.

corresponding theoretically justified approximating relations. Under these conditions the probability of obtaining an unambiguous result is increased many-fold.

2. Results of Simulation of the Correlation of $\Phi_{(2)ij}$. To determine the coefficients d_i, we will write Eq. (4) in the Cartesian coordinate system:

$$F_{xx} = \frac{1}{3} d_2 - d_3 + d_4 \left(b_{xx}^2 + b_{xy}^2 + \frac{2}{3} II\right) + d_4 b_{xx} b^+ + d_5 b_{xx} + \frac{1}{3} d_6 b^+ - d_7 b^+ + d_8 b_8 , \quad (6)$$

$$F_{yy} = \frac{1}{3} d_2 + d_3 + d_4 \left(b_{yy}^2 + b_{xy}^2 + \frac{2}{3} II\right) + d_4 b_{yy} b^+ + d_5 b_{yy} + \frac{1}{3} d_6 b^+ + d_7 b^+ - d_8 b_8 , \quad (7)$$

$$F_{xy} = d_1 + d_2 b^+ + d_3 b^- + 4 d_4 b^+ b_{xy}^2 + 2 d_5 b_{xy}^2 + d_6 b_6 + d_7 b^+ b^- - d_8 b_8 b^- , \quad (8)$$

where $F_{xx} = \Phi_{(2)xx}/4\varepsilon X_1 b_{xy}$; $F_{yy} = \Phi_{(2)yy}/4\varepsilon X_1 b_{xy}$; $F_{xy} = \Phi_{(2)xy}/2\varepsilon X_1$; $X_1 = (K/\varepsilon)(dU_x/dy)$; $b^+ = b_{xx} + b_{yy}$; $b^- = b_{xx} - b_{yy}$; $b_6 = b_{xx}^2 + 2b_{xy}^2 b_{yy}^2$; $b_8 = b_{xy}^2 - b_{xx} b_{yy}$.

To find the coefficients d_i, the distributions $\Phi_{(2)ij}/\varepsilon = F(Y^+)$ presented in [5] are used, which are shown in Fig. 1a. It is known [1] that the value of invariant II changes from 0 to $-1/3$ and of invariant III from -0.01 to $2/27$. Therefore, the dependence of d_i on the invariants II and III can be sought in the form of a series:

$$d_i = d_{i0} + d_{i1} II + d_{i2} III + d_{i3} II\, III + \ldots \quad (9)$$

We note that invariant III is much smaller than invariant II. Therefore, as a first approximation we will use only the first two terms of expansion (9).

From relations (6)-(8) it follows that the functions F_{xx}, F_{yy}, and F_{xy} depend on the value of the coefficients d_α, scalar invariant II, and of the components of the anisotropy tensor b_{ij}. According to Eq. (9), the coefficients d_α ($\alpha = 1-8$) depend in turn on the invariants II and III. Therefore, it is worthwhile to investigate in which way DNS data are connected with these invariants. The values of F_{ij} as functions of invariant II are shown in Fig. 1b. We will pay attention to the behavior of the considered functions in the case of a strongly anisotropic turbulence, when the absolute value of scalar invariant II is increased. In [1] this problem was given special examination. For this purpose, the diagram of the states of turbulence was considered, which showed that in the limiting case of one-dimensional turbulence invariant II tended to the value $-1/3$ and III to $2/27$, whereas the process of redistribution of energy between the components of the tensor of Reynolds stresses ceased. Consequently, the tensor $\Phi_{(2)ij}/\varepsilon \to 0$ and the functions F_{xx}, F_{yy}, and F_{xy} must also tend to zero, when $II \to -1/3$ and $III \to 2/27$.

The character of the dependence of separate functions in Eqs. (6)-(8) on invariant II is shown in Fig. 2. It is seen that some of the distributions presented are approximately similar. Under these conditions there is a great chance of finding several systems of the coefficients d_α, using which it is possible to give a satisfactory description of the term $\Phi_{(2)ij}$. In other words, the problem of an unambiguous selection of the system of coefficients requires a careful and substantiated approach. For this purpose, we will rewrite Eqs. (6) and (7) in the form of their sum and difference:

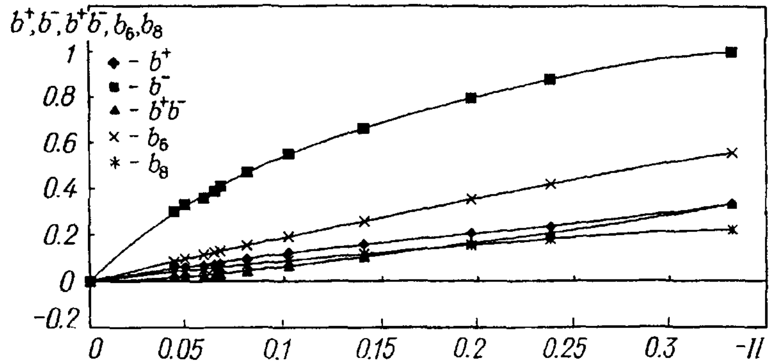

Fig. 2. Dependence of b^+, b^-, b^+b^-, b_6, and b_8 on invariant II.

$$F_{xpy} = d_2 - d_4 II + \left(\frac{3}{2} d_5 + d_6\right) b^+ , \tag{10}$$

$$F_{xmy} = - d_3 + d_4 b^+ b^- + \frac{1}{2} d_5 b^- - d_7 b^+ , \tag{11}$$

where

$$F_{xpy} = \frac{3}{2} (F_{xx} + F_{yy}) ; \quad F_{xmy} = \frac{1}{2} (F_{xx} - F_{yy}) .$$

Here $F_{xpy} \to 0$, $F_{xmy} \to 0$, $F_{xy} \to 0$, when $II \to -1/3$ and $III \to 2/27$. Substituting these conditions into Eqs. (8)-(11), we obtain three additional relations that establish connections between the coefficients $d_{\alpha\beta}$:

$$d_{20} = \frac{1}{3} \left(d_{21} - d_{40} + \frac{1}{3} d_{41} - \frac{3}{2} d_{50} + \frac{1}{2} d_{51} - d_{60} + \frac{1}{3} d_{61}\right) , \tag{12}$$

$$d_{30} = \frac{1}{3} \left(d_{31} + d_{40} - \frac{1}{3} d_{41} + \frac{3}{2} d_{50} - \frac{1}{2} d_{51} - d_{70} + \frac{1}{3} d_{71} + \frac{2}{3} d_{80}\right) , \tag{13}$$

$$d_{11} = 3d_{10} + \frac{2}{3} d_{40} - \frac{1}{9} d_{41} + d_{50} - \frac{1}{3} d_{51} + \frac{4}{3} d_{60} - \frac{4}{9} d_{61} , \tag{14}$$

where

$$d_{10} = 0.2 . \tag{15}$$

Expression (15) follows from the theory of isotropic turbulence. From Eqs. (13) and (14) it is seen that the coefficients d_{10} and d_{11} are established automatically, if the coefficients of relations (10) and (11) are known. Thus, equations written in the form of Eqs. (8), (10), and (11) allow one to solve the problem of determining the system of coefficients $d_{\alpha\beta}$ by stages. First, we find the values of d_2, d_4, and $(3d_5/2 + d_6)$ from the distribution for F_{xpy}. Then we obtain d_3, d_5, d_7, and d_8 from the distributions for the functions F_{xmy} and F_{xy} and the coefficients d_{10} and d_{11} from relations (14) and (15).

We will consider Eq. (10) together with relation (12). This equation contains the coefficients d_2, d_4, d_5, and d_6, each of which, in accordance with series (9), may depend on the scalar invariants II and III. There are DNS data for the function F_{xpy}, invariant II and for the function b^+ to determine these coefficients (see Figs. 2 and 3). Using 10 values of the enumerated functions for 10 points of the boundary layer in a channel and substituting them into relation (10) we obtain 10 linear equations for unknown coefficients. From Fig. 2 it is seen that $b^+ \cong -II$. Therefore, from the data available it is possible to determine only one coefficient d_{20} and two groups:

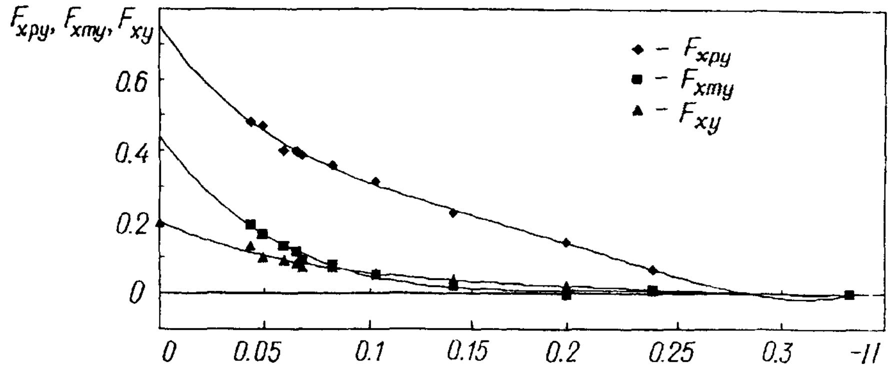

Fig. 3. DNS data for the functions F_{xpy}, F_{xmy}, and F_{xy}.

$(d_{21} - d_{40} - 1.5d_{50} - d_{60})$ and $(d_{41} + 1.5d_{51} + d_{61})$. Here there exist two variants of the description of the data for F_{xpy}:

Variant 1	Variant 2
$d_{20} = 0.48 - 0.49$	$d_{20} = 0.58 - 0.6$
$d_{21} - d_{40} = 1.5d_{50} - d_{60} = 1.45 - 1.55$	$d_{21} - d_{40} - 1.5d_{50} - d_{60} = 3.2 - 3.3$
$d_{41} + 1.5d_{51} + d_{61} = 0$	$d_{41} + 1.5d_{51} + d_{61} = -(4.3 - 4.6)$
accuracy $\delta = 10\%$.	accuracy $\delta = 3\%$.

Thus, the coefficients d_{10} and d_{20} are known, while for determining the remaining 14 values of $d_{\alpha\beta}$ it is possible to write 10 equations for the function F_{xmy} and 10 equations for the function F_{xy}. Moreover, there are three relations (12)-(14) and two relations for the groups $(d_{21} - d_{40} - 1.5d_{50} - d_{60})$ and $(d_{41} + 1.5d_{51} + d_{61})$. Consequently, it remains to determine only nine independent coefficients. This can be done by solving the overdetermined system consisting of 20 linear equations. Analysis of the results of corresponding calculations showed that it is impossible to obtain an unambiguous solution for the system of unknown coefficients, because some of the functions shown in Fig. 2 have similar distributions. For example, in the equation for F_{xmy} there is the scalar invariant II at d_{31} and the function b^+ at d_{70}. Since $b^+ \cong II$, it is possible to find only the difference $(d_{31} - d_{70})$ when solving the system of equations for the coefficients $d_{\alpha\beta}$. A similar situation also occurs when other coefficients are determined.

The above comparatively detailed description of the technique of determining unknown coefficients in the approximation for $\Phi_{(2)ij}$ shows that it is somewhat difficult to obtain an unambiguous result from DNS data for a single specific flow. To determine them, in the present work we will consider a simpler variant of relation (9), when the coefficients d_α are assumed to be constants. This does not mean that they really are independent of the invariants II and III. Possibly, there is such a connection, but here we seek some mean values of the coefficients d_α in that range of scalar invariants, which is observed in a developed channel flow. We note that for a channel flow the range of variation of the scalar invariants II and III is very large, therefore we may hope that the system of empirical coefficients obtained in this way will be universal enough. Thereafter, as the DNS data will appear for other types of flows, this system of coefficients can be refined.

Within the framework of this assumption the coefficients d_{10} and d_{20} are known, the coefficient d_{11} is defined by Eq. (14), and, to determine the remaining six coefficients $d_{30} - d_{80}$, there are two relations (12) and (13) and the value of one group $(-d_{40} - 1.5d_{50} - d_{60})$ obtained earlier by considering the equation for the function F_{xpy}. Consequently, the number of independent coefficients decreases to three, and to determine them there is an overdetermined system of 20 linear equations. It was solved for unknown coefficients. An analysis of the results obtained showed that if the coefficients $d_2 - d_8$ were constants, then the system of the coefficients sought had a single-value solution:

$$d_{10} = 0.2\,,\ d_{11} = -1/3\,,\ d_2 = 1/2\,,\ d_3 = -0.3\,,\ d_4 = 0.2\,,$$
$$d_5 = -1.2\,,\ d_6 = 0.1\,,\ d_7 = -0.7\,,\ d_8 = 0\,. \tag{16}$$

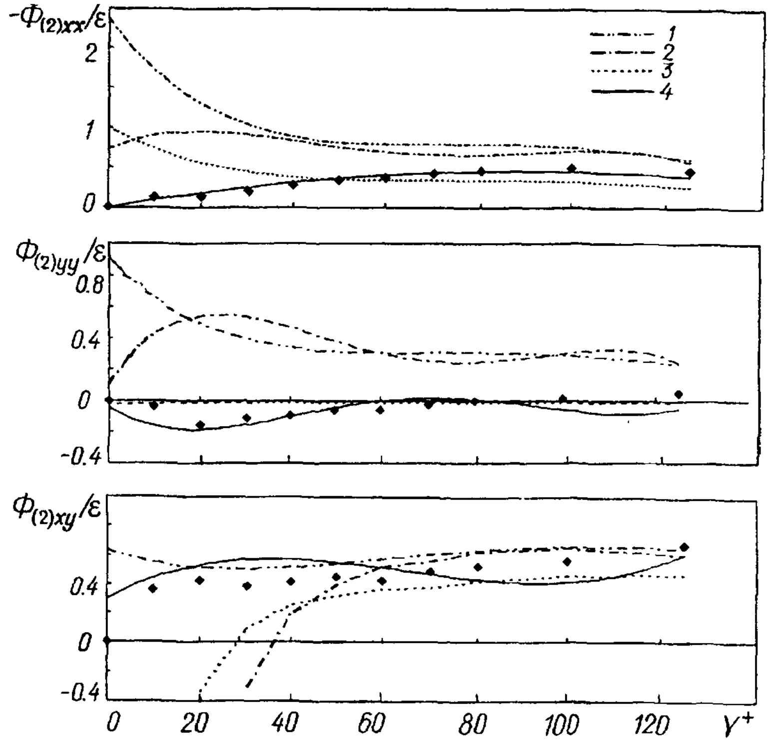

Fig. 4. Comparison of the results of calculations with DNS data for the components of the tensor $\Phi_{(2)ij}/\varepsilon$: 1) formula of [6], 2) [2], 3) [7], 4) calculation by Eq. (4). The dots denote DNS data.

Figure 4 presents a comparison of the DNS data with the results of calculations of $\Phi_{(2)ij}$ by the models of [2, 6, 7] and by formula (4) using the coefficients written in Eq. (16). It is seen from the figure that the models given in [2, 6, 7] satisfactorily describe the components $\Phi_{(2)xx}/\varepsilon$ and $\Phi_{(2)xy}/\varepsilon$ for a low degree of anisotropy of turbulence, which is observed in the axial region of flow, and characterize the distributions of the commponent $\Phi_{(2)yy}/\varepsilon$ quite unsatisfactorily. This indicates that the approximations published earlier for the tensor considered do not take account of certain essential details in the mechanism of the redistribution of energy induced by pressure fluctuations. The models published in [2, 6, 7] must be recognized as inadmissable for describing the distributions of $\Phi_{(2)ij}/\varepsilon$ in the near-wall portion of the boundary layer ($Y^+ < 60$), which is characterized by a high degree of anisotropy of turbulence. The data presented show that the use of the proposed model makes it possible to calculate, with a high enough accuracy, the distributions of the correlation of $\Phi_{(2)ij}$. It is possible to assume that a high degree of description of all three distributions shown in Fig. 4 correctly reflects the mechanism of the pressure fluctuations-induced redistribution of energy between the components of the tensor of Reynolds stresses.

3. Model for the Correlation of $\Phi_{(3)ij}/\varepsilon$. For the first time the question about the necessity to model the term $\Phi_{(3)ij}/\varepsilon$, which takes account of the effect of the wall on the process of the redistribution of energy due to pressure fluctuations, was considered in [6]. The authors used the wall functions that incorporated a specially normalized distance from the investigated point of the flow to the solid wall. However, the DNS data showed unambiguously that the term $\Phi_{(3)ij}/\varepsilon$ was negligibly small over the entire region of the flow up to the layers located in the immediate vicinity of the wall. Due to this, the authors of [8] rejected their own suggestion to use wall functions. As the results of the present work and of [1] showed, the unsatisfactory description of the process of pressure fluctuations-induced redistribution of energy was associated in [2, 6, 7] with the unsatisfactory characteristic of the terms $\Phi_{(1)ij}/\varepsilon$ and $\Phi_{(2)ij}/\varepsilon$, rather than with the effect of the wall. Therefore, we assume

$$\Phi_{(3)ij} = 0\,. \tag{17}$$

4. Results of Simulation of the Correlation of Deformation Rates with Pressure Fluctuations. The total approximation for the correlation of the rates of deformation with pressure fluctuations can be made by relations

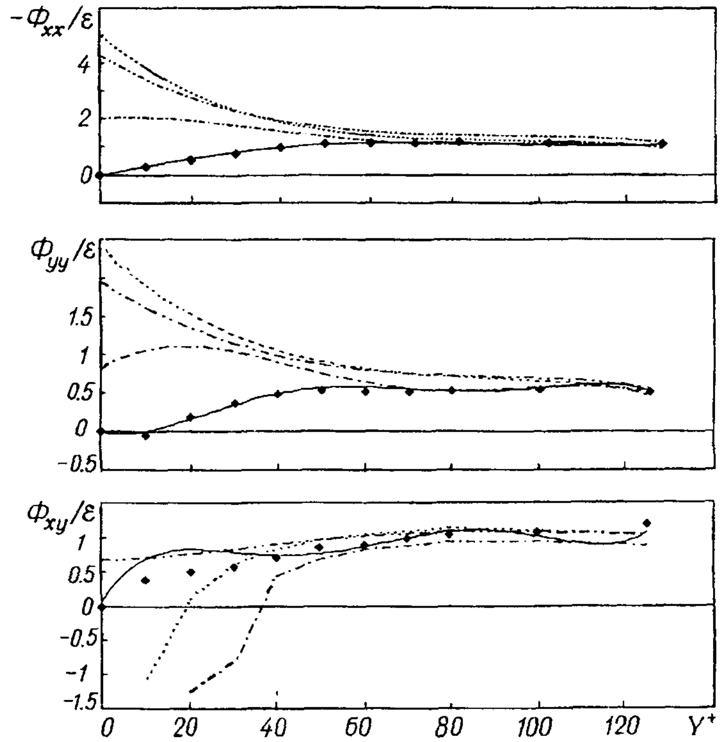

Fig. 5. Comparison of the results of calculation with DNS data for the components of the tensor Φ_{ij}/ε: 1-4) symbols as in Fig. 4. The dots denote DNS data.

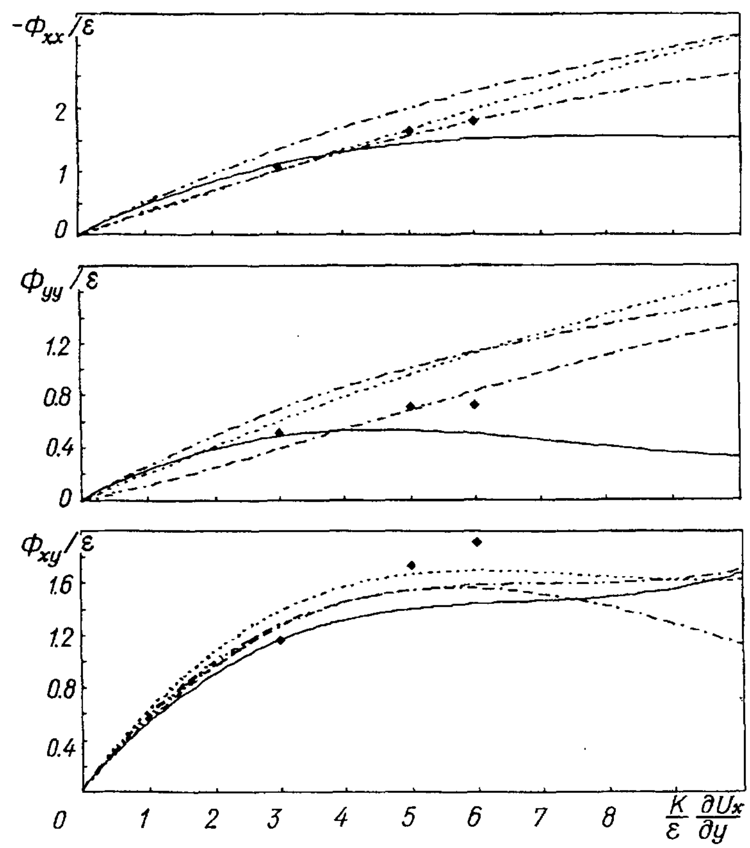

Fig. 6. Comparison of the results of calculation of $\Phi_{(2)ij}/\varepsilon$ with data of measurements of homogeneous turbulence: 1-4) symbols as in Fig. 4. The dots present the data of [9]-[11].

(2), (3), (4), (17) and by the coefficients written in (3) and (16). Figure 5 demonstrates the distributions of the components of the tensor Φ_{ij} across the boundary layer for a developed channel flow. Below we present the values of the mean, for all the components, error in the description of DNS data ($\delta = \nabla\Phi_{(2)ij}/P$), when using the relations and empirical coefficients suggested in the works published:

the model of Launder, Reece, and Rodi [6], $\delta = 47\%$;

the model of Shih, Lumley [2], $\delta = 38\%$;

the model of Speziale, Sarkar, and Gatski [7], $\delta = 48\%$;

model (4), $\delta = 5\%$.

The distributions of the components of the tensor Φ_{ij} for a homogeneous channel flow with a constant shift in velocity are presented in Fig. 6. The values of the normalized velocity gradient, which is the sole parameter that determines this type of flow, are presented as abscissa. The dots denote the data of [9-11]; the curves give the results of calculations by the proposed model and the models of [2, 6, 7]. It is seen from the figures that the models of [2, 6, 7] agree satisfactorily with experimental data for homogeneous flow. This is explained by the fact that the empirical coefficients of the models considered were determined from the results of experiments for homogeneous flow. The unsatisfactory description of the DNS data (Fig. 5), especially in the near-wall region of flow, indicates that these models ignore all the essential mechanisms in the process of redistribution of energy between the components of the tensor of Reynolds stresses. In the proposed model the empirical coefficients were determined on the basis of the DNS data. The results presented in Fig. 6 show that this model is also rather good for describing experimental data for homogeneous flow with a constant shift in velocity.

Thus, on the basis of DNS data a relation was obtained that approximates the correlation of the rates of deformation with a pressure gradient. The approximation has a rather complex form, since it contains terms that are quadratic in the tensor of anisotropy of Reynolds stresses. However, the use of the approximation developed allows one to obtain a many-fold increase in the accuracy with which the process of redistribution of energy due to pressure fluctuations is described, especially in the wall region of flow and in flows with a high degree of anisotropy of turbulence. At the present state of development of the technique of computations the complexity of approximation cannot exert a substantial effect on the time of computations; therefore the results of the work can be used in numerical calculations based on the models of 2nd-order turbulence.

NOTATION

$R_{ij} = \langle u_i u_j \rangle$, single-point correlation of velocity fluctuations; $F_{ij} = (\langle f_i u_j \rangle + \langle f_j u_i \rangle)/\rho$, the term representing the production of the energy of turbulence caused by the effect of external force; $P_{ij} = -(R_{ik}U_{j,k} + R_{jk}U_{i,k})$, production of the energy of turbulence by the mean velocity gradient; $\Phi_{ij} = (1/\rho) < p(u_{i,j} + u_{j,i}) >$, correlation of the rates of deformation with pressure fluctuations; $D_{ij} = -[\langle u_i u_j u_k \rangle + \frac{1}{\rho}(\langle pu_i \rangle \delta_{jk} + \langle pu_j \rangle \delta_{ik} - \nu \langle u_i u_j \rangle_{,k}]_{,k}$, diffusion term; $\varepsilon_{ij} = \nu \langle u_{i,k} u_{j,k} \rangle$, dissipation term; u_i, f_i, p, fluctuations of velocity, external force, and pressure, respectively; ρ, density; ν, kinematic viscosity; δ_{ij}, Kronecker symbol; sharp brackets denote averaging; the comma in front of the subscript denotes differentiation; $b_{ij} = \frac{R_{ij}}{2K} - \frac{1}{3}\delta_{ij}$, tensor of the anisotropy of Reynolds stresses; $d_{ij} = \frac{\varepsilon_{ij}}{\varepsilon} - \frac{1}{3}\delta_{ij}$, tensor of the anisotropy of the rate of dissipation of Reynolds stresses; K, kinetic energy of turbulence; P, production of the energy of turbulence; ε, rate of dissipation of the energy of turbulence; $II = -b_{ik}b_{kj}b_{ij}/2$, $III = b_{ik}b_{kj}b_{ij}/3$, scalar invariants of the anisotropy of the Reynolds stresses of the 2nd and 3rd orders; $F = 1 + 9II + 27III$, scalar invariant that determines the degree of anisotropy of turbulence; $\Phi_{(1)ij}$, tensor that describes the approach of turbulence to an isotropic state; $\Phi_{(2)ij}$, "fast portion" in the correlation of the rate of deformation with pressure fluctuations; $\Phi_{(3)ij}$, term taking account of the effect of the wall on the redistribution of the energy of turbulence; d_α and $d_{\alpha\beta}$, coefficients of approximation; $Y^+ = yu_0/\nu$, dimensionless distance from the wall; u_0, dynamic velocity on the wall.

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